Application of Monte Carlo method in the computer system for valuation of exotic option contracts

Zastosowanie metody Monte Carlo w informatycznym systemie wyceny egzotycznych kontraktów opcyjnych

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Treść. W artykule prezentowana jest propozycja komputerowego systemu obliczania finansowych instrumentów pochodnych – opcji egzotycznych i hybrydowych. Handel takimi produktami odbywa się na rynku pozagiełdowym (OTC) i często są to produkty tworzone na zlecenie. Wartości pewnych rodzajów opcji egzotycznych i hybrydowych nie można wyliczyć tradycyjnymi analitycznymi i numerycznymi metodami. W takich przypadkach warto użyć metody Monte-Carlo do wyceny rzeczywistej wartości instrumentów finansowych. W artykule przedstawiony został sposób obliczania ceny przykładowej opcji egzotycznej za pomocą metody MC. System komputerowy oparty o MC mógłby służyć do wspomagania decyzji inwestycyjnych dotyczących egzotycznych kontraktów opcyjnych.

Słowa kluczowe: metody Monte-Carlo, systemy wspomagania decyzji, opcje egzotyczne i hybrydowe, inżynieria finansowa, wycena kontraktów opcyjnych

Abstract. This paper presents a proposal for a computer system of calculation of financial derivatives - exotic and hybrid options. The trade of such products takes place on the OTC market and these products are often made and tailored on demand. The values of certain types of exotic and hybrid options cannot be calculated with traditional analytical and numerical methods. In such cases, it is worth to use the Monte-Carlo method for the valuation of the real value of financial instruments. This paper presents an example of calculating the price of exotic options using the MC method. The computer system based on MC could be used to support investment decisions regarding exotic option contracts.

Keywords: Monte-Carlo methods, decision support systems, exotic and hybrid options, financial engineering, valuation of option contracts

1. Introduction

Since the second half of the twentieth century the rapid development of applications for option contracts has been noted. These products make it possible to achieve above-average investment returns regardless of economic conditions. They can be based on various underlying assets such as stocks, currencies, indices and commodities. According to the theory [11, 14], an option is a contract between the buyer and a seller that gives the buyer the right (but not the obligation) to call or put underlying asset at a specified time in the future at a preset price, in exchange for a fee called the option premium. The option price is dependent on the value of the underlying (base) assets. Having the option gives you the right, but no obligation, and therefore the option holder can exercise this right, when it is profitable for him. The exercise of the right is called the exercise of option or its settlement. The holder of the American option exercises it at any time from the moment of purchase to the date of termination. European option can only be exercised on the option's expiry date. So in the case of the European option the exercise and expiration dates are the same. American and European types of option contracts are referred to as standard or vanilla ones. Any other option contracts are called exotic or hybrid.

Financial institutions seeking more efficient and effective management methods of capital in the financial markets began to commonly use option contracts with the arrival of the first analytical [1] and discrete [6] models of the evaluation of these financial products. There are many reasons why exotic and hybrid options are attractive to investors. Among the most important, one may mention that the usual price of exotic and hybrid options is lower than the one of the standard options, and in most cases they give similar and adequate investment and security as vanilla options (American and European). Besides, new contracts created also expand risk management capabilities. However, technological development, reflected by the increase in computing power, can solve complex numerical problems related to the valuation of financial products in real time.

Larger and more diverse requirements of investors lead to the formation of successive derivatives. Among the recently introduced contracts, significant and increasingly important role is played by exotic and hybrid options. These are products of a more complex structure than the standard options. Trading in these instruments can be done at Over The Counter market (OTC) and/or they can be tailored to a specific investor's needs. In the case of valuation of ta-

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ilored instruments, frequently important is the efficiency of the preparation of a valuation model [5, 16]. Analytical methods cannot be used to describe the valuation formula of all the exotic option contracts. And given that certain types of exotic options and hybrids cannot be measured by any traditional methods, it makes sense in such cases to use the Monte Carlo simulation to evaluate the price of the financial instruments concerned.

2. Monte Carlo method for valuation of option contracts

In this chapter the effective MC method [8, 16] of valuation of exotic options with a sample valuation of a simple Lookback type contract will be presented. Monte Carlo method may be used in a variety of stochastic processes, even if the natural logarithm of the value of the basic instrument behaves in accordance with the geometric Brownian motion:

$$dS=\left(1+\mu S+\frac{1}{2}\sigma^2 S\right)dt+\sigma S dZ$$

Using Monte Carlo methods for the valuation of options was first presented by Boyle [2, 10, 11]. MC is usually used when there are no analytical and discrete models. That is, when the price of the basic instrument S is given by:

$$S+dS=Se^{\left(\mu-\sigma^2/2\right)dt+\sigma dZ}$$

where dZ is the differential of the Wiener process [4, 12], with a standard deviation σ equal to one and the average value (drift) μ equal to zero. In order to simulate the process, we split the timeline into a finite number of n intervals of the following ends $t_0, t_1, t_2, \ldots, t_n$ away from each other of Δt symbol,

$$S+\Delta S=Se^{\left(\mu-\sigma^2/2\right)\Delta t+\sigma\varepsilon_t\sqrt{\Delta t}}$$

where ΔS is the growth rate of S price in a given period of time Δt, and ε_t are independent random values generated from the standard normal distribution $N(0,1)$. Most programming languages have built-in function that returns a random value from a normal distribution. If, however, there are only random Z numbers from 0 to 1, they have to be converted into a random value of the normal distribution. Subsequent independent S_n values are calculated using the following rule [10]:
, for m=1…n

$$S_m=S_{m-1}\,e^{\left(\mu-\sigma^2/2\right)\left(t_m-t_{m-1}\right)+\sigma\varepsilon_t\sqrt{t_m-t_{m-1}}}$$

Assuming that the payoff function of the W option depends only on the value of the basic S_n one can find the formula for the option price C for a single trajectory (Figure 1) of the basic instrument prices $S_0, S_1, S_2, \ldots, S_n$:

$$C=e^{-rT}W\left(S_0,S_1,S_2,\ldots,S_n\right)$$

but after the performance of k experiments (Figure 2), the option premium based on average is equal to:

$$C=e^{-rT}\frac{1}{k}\sum_{eks=1}^{k}W\left(S_{t0}^{eks},S_{t1}^{eks},S_{t2}^{eks},\ldots,S_{tn}^{eks}\right)$$

It needs to be borne in mind that for each type of option the payment function W is calculated differently. Each of the trajectories in Figure 2 corresponds to one experiment from the formula.

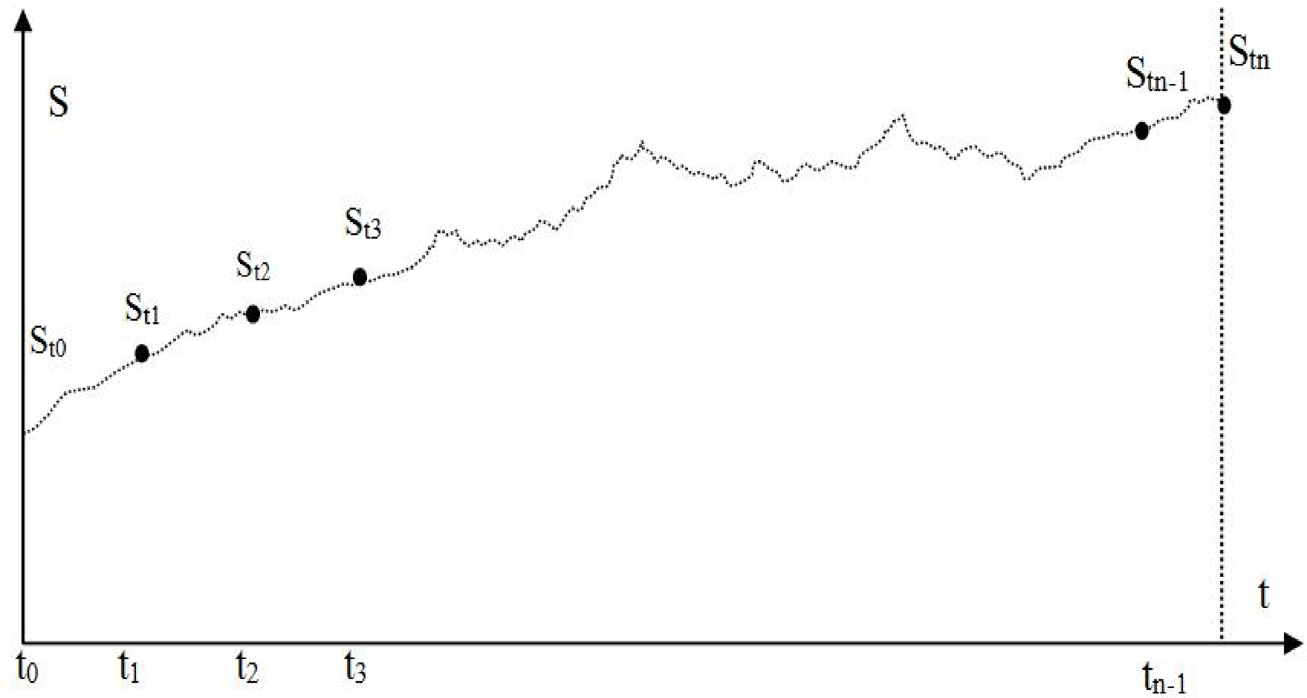

Fig. 1. A single price trajectory of underlying instrument S.
Rys. 1. Pojedyncza trajektoria cen S instrumentu podstawowego.

Source: own research

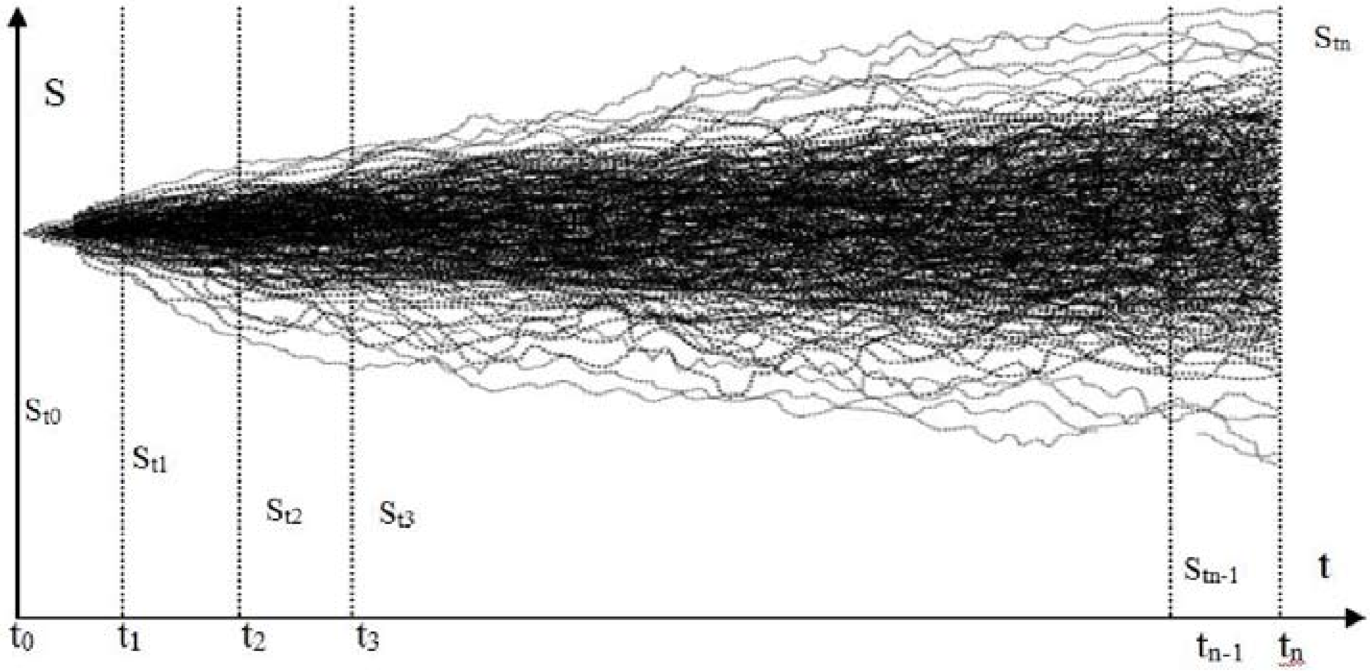

Fig. 2. Random trajectories of the underlying asset price S after performance of k experiments.
Rys. 2. Losowe trajektorie cen instrumentu bazowego S po przeprowadzeniu k eksperymentów.

Source: own research, [17]

Classic method of valuation, proposed in the original version, using the Monte Carlo simulation is relatively ineffective. Satisfactory option premium valuation accuracy is achieved only after generating at least ten thousand *k* trajectories. In addition, for up to 10-fold increase in the accuracy of calculations it is required to carry out up to 100 times more experiments [8]. A number of techniques to speed up the described method was developed. The most often used methods are applied via the reduction of variance [3, 11]. These include:

- antithetic variates
- stratified sampling
- importance sampling
- control variates
- moment matching.

One of the most practical technique is the antithetic variates method which consists for every sample (primary) path obtained additional symmetric antithetic path. This technique has two advantages. It decreases the variance of the sample paths, improving the accuracy. Also it reduces the number of samples to be generated to obtain N paths. Figure 3 below shows exemplary paths.

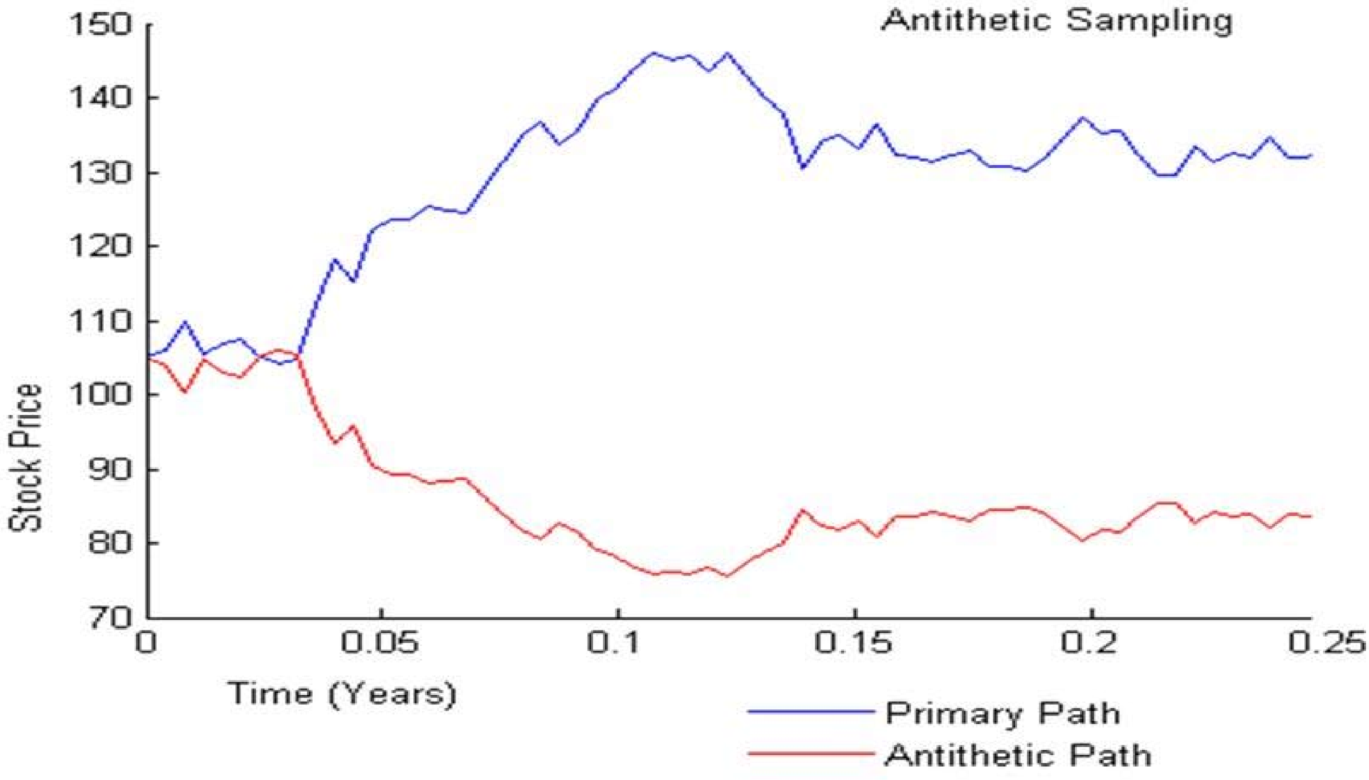

Fig. 3. Symmetric primary and antithetic sample paths.
Rys. 3. Symetryczne ścieżki próbki pierwotnej i antytetycznej.

Source: [18], own research

3. Applying the MC for the Lookback option valuation

We will now consider an effective method of valuation using Monte Carlo method for Lookback option – popular exotic contract. There are two basic types of options; call and put. The value of the payment from option depends upon the exercise price *X*. The payment function for the Lookback contract is also subject to the average S_A price of the underlying instrument. The following table shows the functions of payment *W* of Lookback options.

Tab. 1. Payment functions of Lookback options.
Tab. 1. Funkcje wypłaty opcji wstecznych.

Type:	Payment function
Lookback floating-strike call	$W_{call} = \max\left(0, S(T_{Final}) - \min_{t_i} S(t_i)\right)$
Lookback floating-strike put	$W_{put} = \max\left(0, \min_{t_i} S(t_i) - S(T_{Final})\right)$

Source: own research

According to the approach proposed by Broadie and Glasserman [3] in order to perform a single experiment, one may needs to find values at t_i. The examples below shows the formula of calculating the value of the option at expiry (payoff function) using MC method:
where:

T_0 – option start date
T_{Final} – final payment date

$$W_{call} = \max\left(0, \frac{S(T_{Final}) - \min_{t_i} S(t_i)}{S(T_0)}\right)$$

$$W_{put} = \max\left(0, \frac{\max_{t_i} S(t_i) - S(T_{Final})}{S(T_0)}\right)$$

$t_1, t_2, \ldots, t_m$ with $T_0 \le t_1 < t_2 < \ldots < t_m \le T_{Final}$ - Lookback option dates
$S(t)$ denotes the price of the underlying at time t.
After performing *k* experiments and discounting W_{call} (W_{put}) to the current value, one can obtain option price:

$$C_{call} = e^{-rT} \frac{1}{k} \sum_{eks=1}^{k} W_{call}^{eks}$$

$$C_{put} = e^{-rT} \frac{1}{k} \sum_{eks=1}^{k} W_{put}^{eks}$$

When setting the value of the option, usually the price sensitivity factors are calculated at the change in the factors that determine the price. The factors that affect the price of an option are: the price of the underlying instrument *S*, exercise price *X*, volatility of the underlying instrument σ, the length of time for the expiration date *t,* and the risk-free interest rate *r*. Accordingly delta, gamma, lambda, theta and ro coefficients indicate how the option price will change when the factors affecting the price of an option will change the unit of measurement. Sensitivity coefficients are very valuable information, from an investor's perspective. Therefore, they are an essential part of the system of valuation. Detailed description of methodology for determination of the sensitivity coefficients in the MC method is beyond the scope of this paper. More on this topic can be found in the development by Michael C. Fu and Jian Qiang Hu [9].
The exemplary valuation results are presented below. N is the number of simulations. There is a minimum difference in results when N=1000 and N=5000. It shows that the method performs well comparing to Internet option calculators. It could be used for valuation of real market options.

Tab. 2. European lookback call. Parameters: S=100, X=100, σ=0,2, t=0,5, r=0,03.
Tab. 2. Europejska wsteczna opcja kupna Parametry S=100, X=100,σ=0,2, t=0,5, r=0,03.

N	Option value
100	9.251
500	9.524
1000	9.653
5000	9.701

4. The computer system for evaluating the value of exotic options

According to Turban, decision support systems [15, 7, 13, 16] are designed for decision-makers at various levels of the organization and deal with the semi-structuralized decisions. A financial analyst meets such problem during the decision making process, whether the timing is good to invest and call/put option contracts. Automated data processing, leading to a clear investment decision is not possible. If the method were possible to be expresses as an algorithm, the decision-making process would be structuralized and could be solved automatically. However, we deal with the problem of poor or poorly structuralized, in which there is no algorithm clearly indicating the correct choice. Only through guidance such as the theoretical va-

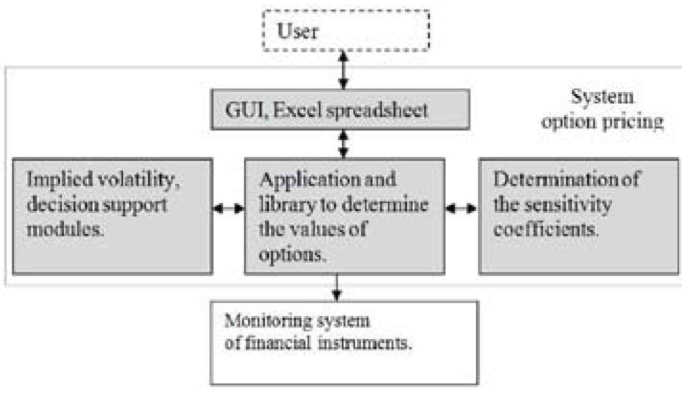

Fig. 4. Schematic diagram of the exotic options valuation system.
Rys. 4. Schemat poglądowy systemu wyceny opcji egzotycznych.
Source: own research

lue of the option and the sensitivity coefficients one can support decision-making processes.
The system design should focus both on improving the currently available solutions and larger offer of available instruments. Schematic diagram of the system is shown in Figure 4.
The data is entered into the system by the user into a spreadsheet forming a graphical user interface. It is worth mentioning that, spreadsheets, and especially Excel, are standard among financial analysts. GUI in the sheet should be based on an intuitive approach and prevent invalid data entry. For example, entry values for the options with unrealistic data such as incorrect dates must be rejected. After filtration and preparation of the input data with VB code, they are sent from the spreadsheet to the evaluation application. The designated value of the option can then be used to calculate the sensitivity coefficients. Additional functionality of the system, not described in this document due to the extensiveness, should include: determining implied volatility [3, 11], decision support modules, and preparation of charts of payoff function. All results are sent to the spreadsheet. The theoretical value of financial instruments and the sensitivity coefficients can be transferred to an external real-time system controlling open positions in the contracts.
Decision support module role is to indicate potential investment opportunities. For instance compare the market price of the option contract with the calculated theoretical price, track changes in the sensitivity coefficients (delta, gamma, theta, lambda, rho). Such information helps the investor to decide whether to buy or sell options.
Currently, in the software market, one can find a lot of tools that allow easy and rapid application development (eg, .Net). Application for evaluation of the options and the sensitivity coefficients should be formed by the fastest object oriented programming languages and also the fourth-generation languages (such as C++, C#). Their biggest advantage is the ability to create complex libraries without writing too much code in an automated way, based on standard components and features.

5. Conclusions

In order to create a practical system for the valuation of exotic options one must meet the high requirements set by the investors, especially regarding the speed and accuracy of calculations. Technological advances and the growing capabilities of computers allow the use of real-time pricing scheme for valuing options contracts based on the Monte Carlo methods. Presented valuation system is designed in such a way as to ensure the proper evaluation of financial products and enable efficient decision support for investment in both market and OTC options.
The biggest advantage of this system is to provide an uncomplicated prospect of adding further complex financial products algorithms. In most cases, for the introduction of a new product it will only be necessary to prepare and implement contract payment function. The payment function can use a common MC generator and associated libraries. A wide range of financial instruments available in the library will meet the growing demands of investors. The system should also have security mechanisms against the introduction of incorrect data, calculate implied volatility, sensitivity coefficients and have a decision support module. The application on the Polish market due to its abilities and novelty could be a practical and unique tool supporting various investment in options.

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